8. Appendix

8.1 Survey1

Tools for Learning Logic - Survey

This survey aims to collect some data for a project investigating how the software tool Jape might help you in the "Introduction to Logic" course. The data is kept completely confidential. The first section is concerned with collecting some background information. The second section asks about your expectations of studying computer science. The final section is to do with reasoning.

We appreciate your co-operation with us in our research. Thank you for your help,

James Aczel & Pat Fung, Open University

Section 1: Background

Male ☐ Female ☐ **Please tick a box**

Age: Under 25 ☐ 25+ ☐ **Please tick a box**

Family Name: _____ *First Name:* _____

In which subjects do you have A-level (or equivalent) passes?

For what degree are you registered?

Please tick one box only

- computer science ☐
- computer science & maths ☐
- maths & computer science ☐
- computer science & linguistics ☐
- computer science & business ☐
- other ☐ *If other, please specify:* _____

Page 1

Survey1 (continued)

How much computer programming have you done before starting this course?

none at all	☐
a little	☐
a fair amount	☐
a lot	☐

Please tick one box only

Before you came to QMW did you have the use of a computer at home?

Yes ☐ No ☐ **Please tick a box**

If so, for what did you most use it?

word processing or DTP	☐
games	☐
spreadsheet	☐
programming	☐
graphics	☐
internet, web or email	☐
database or reference	☐
accountancy or finance	☐
other	☐

Please tick all that apply

If other, please specify:

_____

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Survey1 (continued)

Section 2: Questionnaire

What were the most important factors in your choosing to study a degree that includes computer science?

In the first column, tick ALL the important factors.

In the second column, tick the MOST important factor. Tick ONE ONLY please.

learning to program	☐	☐
following an interest	☐	☐
financial rewards	☐	☐
learning to think logically	☐	☐
learning rigorous methods	☐	☐
stable job	☐	☐
pressure from parents	☐	☐
couldn't think of anything else to study	☐	☐
friends also studying it	☐	☐
other	☐	☐

If other, please specify: _____

What is the value of the "Introduction to Logic" course, as you see it?

How difficult are you expecting the "Introduction to Logic" course to be?

Please tick one box only

- difficult ☐
- fairly difficult ☐
- fairly easy ☐
- easy ☐

How interesting are you expecting the "Introduction to Logic" course to be?

Please tick one box only

- interesting ☐
- fairly interesting ☐
- fairly uninteresting ☐
- uninteresting ☐

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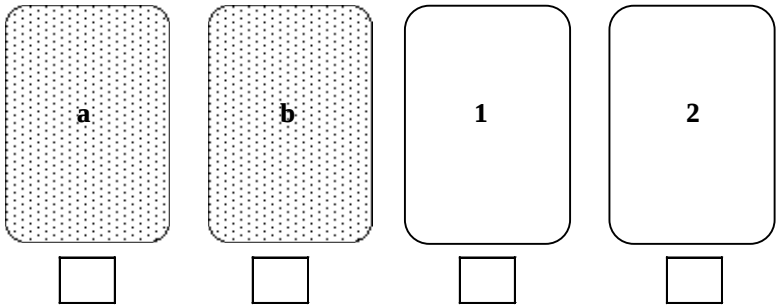
Survey1 (continued)

Section 3: Reasoning

Question One

A special pack of cards is being used. On each card, there is a number on the face, and a letter on the back. You see these four cards on a table:



Tick the box below each card or cards that you think must be turned over to decide whether the following statement is true:

"Every card with a vowel on the back has an even number on the face".

Question Two

(a) If you are given the sum and difference of any two numbers, can you always find out what the numbers are?

Yes ☐ No ☐ **Please tick a box**

Justify your answer

(b) For all integers x and y, if (x + y) can be divided exactly by 3, then x can be divided exactly by 3 and y can be divided exactly by 3. Is this true or false?

True ☐ False ☐ **Please tick a box**

Justify your answer

Survey1 (continued)

Question Three

(a) Is the following argument valid?

If a person is an unmarried man, he is a bachelor.
Julian Clary is a bachelor.
Therefore Julian Clary is an unmarried man.

Yes ☐ No ☐ **Please tick a box**

Please try and show how you got your answer

(b) Is the following argument valid?

All men are mammals.
All mammals can produce milk for their young.
Therefore all men can produce milk for their young.

Yes ☐ No ☐ **Please tick a box**

Please try and show how you got your answer

(c) Is the following argument valid?

Some insects have wings.
All flying creatures have wings.
Therefore some insects can fly.

Yes ☐ No ☐ **Please tick a box**

Please try and show how you got your answer

Survey1 (continued)

Question Four

(a) Judy thinks of a number. She multiplies it by 5, adds 12 to the result, subtracts her original number, and divides by 4. She notices that the final result is 3 more than her original number. Does this always happen, whatever the original number?

Yes ☐ No ☐ **Please tick a box**

Please try and show how you got your answer

(b) Write an equation to represent the statement:
'At this university there are 15 times as many students as professors.'
Use S for the number of students and P for the number of professors.

Answer:

Question Five

In arithmetic multiplication "dominates" addition. This means that $2 + 3 \times 5$ is interpreted as $2 + (3 \times 5)$. Once we've inserted missing brackets, we can identify the "principal operator" of the formula: it's the only one outside the brackets. For example, in $(2 + 3) \times 5$ it's $\times$. In $4 \times 6 + 3 \times 2$, we can insert the missing brackets and write $(4 \times 6) + (3 \times 2)$. So the principal operator is +.

The order of operations is, in descending order of dominance:
brackets, exponentiation (as in 8^3), negation (as in $^{-}6$), $\div$, $\times$, +, –

(a) Insert missing brackets in $7 - 5 \times 4 + 8^3 \div 2 - 9 + 2$ _____

(b) Identify the principal operator in $15 - 9 \times 4$ _____

(c) Identify the principal operator in $8^2 \times {}^{-}6$ _____

(d) Identify the principal operator in $(a + 5) \div b^{c+2} - 1$ _____

8.2 ItL Jape Conjectures

Implication

1. P, P → Q ⊢ Q
2. P → Q, Q → R , P ⊢ R
3. P → (Q → R), P → Q, P ⊢ R
4. P → Q, Q → R ⊢ P → R
5. P → (Q → R) ⊢ Q → (P → R)
6. P → (Q → R) ⊢ (P → Q) → (P → R)
7. P ⊢ Q → P
8. P → (Q → P)
9. P → Q ⊢ (Q → R) → (P → R)
10. P → (Q → (R → S)) ⊢ R → (Q → (P → S))
11. (P → (Q → R)) → ((P → Q) → (P → R))

Conjunction

12. P, Q ⊢ P ∧ Q
13. P ∧ Q ⊢ P
14. P ∧ Q ⊢ Q
15. (P ∧ Q) → R ⊢ P → (Q → R)
16. P → (Q → R) ⊢ (P ∧ Q) → R

Disjunction

17. P ⊢ P ∨ Q
18. Q ⊢ P ∨ Q
19. P ∨ Q ⊢ Q ∨ P
20. Q → R ⊢ (P ∨ Q) → (P ∨ R)
21. P ∨ P ⊢ P
22. P ⊢ P ∨ P
23. P ∨ (Q ∨ R) ⊢ (P ∨ Q) ∨ R
24. (P ∨ Q) ∨ R ⊢ P ∨ (Q ∨ R)
25. P ∧ (Q ∨ R) ⊢ (P ∧ Q) ∨ (P ∧ R)
26. (P ∧ Q) ∨ (P ∧ R) ⊢ P ∧ (Q ∨ R)
27. P ∨ (Q ∧ R) ⊢ (P ∨ Q) ∧ (P ∨ R)
28. (P ∨ Q) ∧ (P ∨ R) ⊢ P ∨ (Q ∧ R)
29. P → R, Q → R ⊢ (P ∨ Q) → R

Negation

30. ¬¬P → P
31. P ⊢ ¬ ¬P
32. P → Q ⊢ ¬Q → ¬P
33. ¬Q → ¬P ⊢ P → Q
34. P ∨ ¬P
35. P ∨ Q ⊢ ¬(¬P ∧ ¬Q)
36. ¬(¬P ∧ ¬Q) ⊢ P ∨ Q
37. P ∧ Q ⊢ ¬(¬P ∨ ¬Q)
38. ¬(¬P ∨ ¬Q) ⊢ P ∧ Q
39. ¬(P ∨ Q) ⊢ ¬P ∧ ¬Q
40. ¬P ∧ ¬Q ⊢ ¬(P ∨ Q)
41. ¬(P ∧ Q) ⊢ ¬P ∨ ¬Q
42. ¬P ∨ ¬Q ⊢ ¬(P ∧ Q)
43. ¬(P ∧ ¬P)
44. Q → P, P → R ⊢ Q → R
45. (P → Q) ∨ (Q → P)
46. P ∧ ¬P ⊢ Q
47. ((P → Q) → P) → P

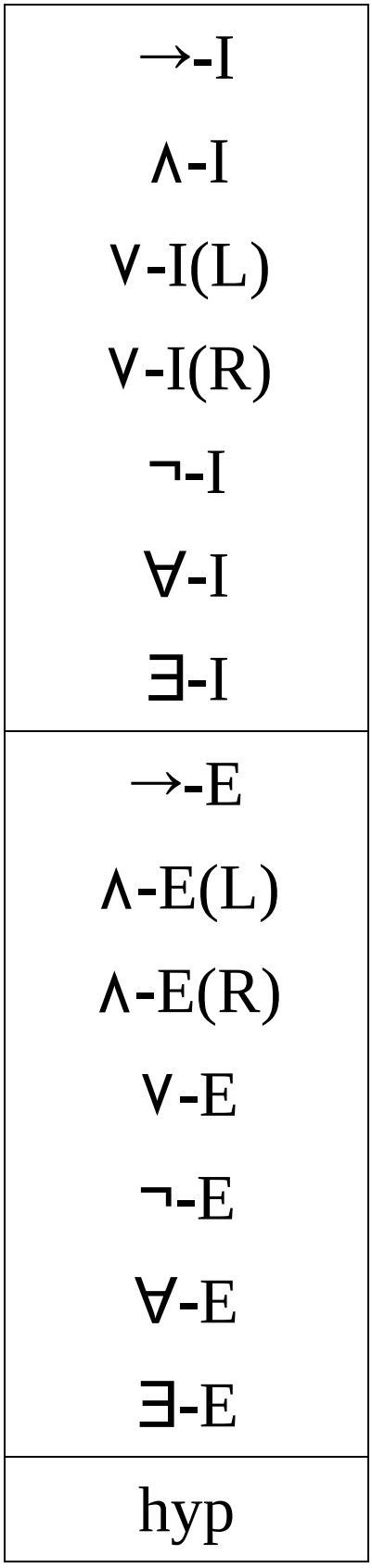
Quantifiers

48. var c, P(c), ∀x.(P(x) → Q(x)) ⊢ Q(c)
49. ∀x.(P(x) → Q(x)) ⊢ ∀x.P(x) → ∀x.Q(x)
50. ∀x.(P(x) → Q(x)), ∀x.(Q(x) → R(x)) ⊢ ∀x.(P(x) → R(x))
51. ∀x.P(x) ∧ ∀x.Q(x) ⊢ ∀x.(P(x) ∧ Q(x))
52. ∀x.(P(x) ∧ Q(x)) ⊢ ∀x.P(x) ∧ ∀x.Q(x)
53. ∀x.(P(x) → Q(x)), ∃x.P(x) ⊢ ∃x.Q(x)
54. ∃x.(P(x) ∧ Q(x)) ⊢ ∃x.P(x) ∧ ∃x.Q(x)
55. ∃x.P(x) ∨ ∃x.Q(x) ⊢ ∃x.(P(x) ∨ Q(x))
56. ∃x.(P(x) ∨ Q(x)) ⊢ ∃x.P(x) ∨ ∃x.Q(x)
57. var c, ∀x.P(x) ⊢ ∃x.P(x)
58. ∀x.P(x) ⊢ ¬(∃x. ¬P(x))
59. ¬(∃x. ¬P(x)) ⊢ ∀x.P(x)
60. ∃x.P(x) ⊢ ¬(∀x. ¬P(x))
61. ¬(∀x. ¬P(x)) ⊢ ∃x.P(x)
62. ¬(∀x.P(x)) ⊢ ∃x. ¬P(x)
63. ∃x. ¬P(x) ⊢ ¬(∀x.P(x))
64. ¬(∃x.P(x)) ⊢ ∀x. ¬P(x)
65. ∀x. ¬P(x) ⊢ ¬(∃x.P(x))

False

66. ∀x.P(x) ⊢ ∃x.P(x)
67. P(c), ∀x.(P(x) → Q(x)) ⊢ Q(c)
68. var c, Q(c) ⊢ ∀x.(P(x) → Q(x))
69. ∀x.P(x) → ∀x.Q(x) ⊢ ∀x.(P(x) → Q(x))
70. ∃x.P(x) ∧ ∃x.Q(x) ⊢ ∃x.(P(x) ∧ Q(x))

8.3 The rules menu in ItL Jape



8.4 Jape usage data for the conjectures used in the Observational Study

Conjecture Number	Conjecture	Topic	Number of students attempting	Number of successful students	% of attempting students who are successful	Number of proof attempts	Number of successful proof attempts	% of proof attempts that are successful	Number of proof attempts per student	Time spent on proof attempts (mins)	Time spent on successful proofs (mins)	% of time that is spent on successful proofs	Time per proof attempt (mins)	Time per successful proof (mins)	Number of proof attempts per hour	Number of successful proofs per hour	Time per student on proof attempts (mins)	Time per student on successful proofs (mins)
1	P, P→Q ⊢ Q	Implication	113	111	98	197	180	91	1.7	112	85	76	0.6	0.5	105	96	1.0	0.8
2	P→Q, Q→R, P ⊢ R	Implication	112	110	98	173	157	91	1.5	117	112	96	0.7	0.7	89	80	1.0	1.0
3	P→(Q→R), P→Q, P ⊢ R	Implication	111	107	96	161	146	91	1.5	188	184	98	1.2	1.3	51	47	1.7	1.7
4	P→Q, Q→R ⊢ P→R	Implication	107	107	100	158	141	89	1.5	334	292	87	2.1	2.1	28	25	3.1	2.7
5	P→(Q→R) ⊢ Q→(P→R)	Implication	104	100	96	152	134	88	1.5	340	317	93	2.2	2.4	27	24	3.3	3.0
6	P→(Q→R) ⊢ (P→Q)→(P→R)	Implication	99	94	95	138	119	86	1.4	308	286	93	2.2	2.4	27	23	3.1	2.9
7	P ⊢ Q→P	Implication	94	94	100	134	122	91	1.4	36	31	86	0.3	0.3	222	202	0.4	0.3
8	P→(Q→P)	Implication	91	88	97	135	113	84	1.5	71	57	79	0.5	0.5	114	95	0.8	0.6
9	P→Q ⊢ (Q→R)→(P→R)	Implication	89	84	94	121	108	89	1.4	143	115	81	1.2	1.1	51	45	1.6	1.3
10	P→(Q→(R→S)) ⊢ R→(Q→(P→S))	Implication	87	85	98	120	108	90	1.4	226	208	92	1.9	1.9	32	29	2.6	2.4
11	(P→(Q→R))→((P→Q)→(P→R))	Implication	82	74	90	112	90	80	1.4	161	142	88	1.4	1.6	42	34	2.0	1.7
12	P, Q ⊢ P∧Q	Conjunction	74	73	99	97	88	91	1.3	61	50	82	0.6	0.6	95	87	0.8	0.7
13	P∧Q ⊢ P	Conjunction	75	73	97	99	92	93	1.3	70	68	97	0.7	0.7	85	79	0.9	0.9
14	P∧Q ⊢ Q	Conjunction	71	69	97	102	88	86	1.4	38	26	69	0.4	0.3	163	141	0.5	0.4
15	(P∧Q)→R ⊢ P→(Q→R)	Conjunction	76	67	88	120	87	73	1.6	311	224	72	2.6	2.6	23	17	4.1	2.9
16	P→(Q→R) ⊢ (P∧Q)→R	Conjunction	67	60	90	97	77	79	1.4	210	173	83	2.2	2.2	28	22	3.1	2.6

Figure 145: Usage figures for Implication and Conjunction (conjectures 1-16)

Conjecture Number	Conjecture	Topic	Number of students attempting	Number of successful students	% of attempting students who are successful	Number of proof attempts	Number of successful proof attempts	% of proof attempts that are successful	Number of proof attempts per student	Time spent on proof attempts (mins)	Time spent on successful proofs (mins)	% of time that is spent on successful proofs	Time per proof attempt (mins)	Time per successful proof (mins)	Number of proof attempts per hour	Number of successful proofs per hour	Time per student on proof attempts (mins)	Time per student on successful proofs (mins)
17	P ⊢ P∨Q	Disjunction	61	58	95	85	74	87	1.4	53	47	88	0.6	0.6	96	84	0.9	0.8
18	Q ⊢ P∨Q	Disjunction	59	59	100	77	76	99	1.3	52	52	99	0.7	0.7	89	88	0.9	0.9
19	P∨Q ⊢ Q∨P	Disjunction	58	55	95	86	67	78	1.5	242	168	69	2.8	2.5	21	17	4.2	2.9
20	Q→R ⊢ (P∨Q)→(P∨R)	Disjunction	57	49	86	87	59	68	1.5	377	209	55	4.3	3.5	14	9	6.6	3.7
21	P∨P ⊢ P	Disjunction	50	48	96	62	58	94	1.2	37	31	84	0.6	0.5	100	93	0.7	0.6
22	P ⊢ P∨P	Disjunction	50	49	98	64	59	92	1.3	18	18	97	0.3	0.3	209	193	0.4	0.4
23	P∨(Q∨R) ⊢ (P∨Q)∨R	Disjunction	52	35	67	119	45	38	2.3	638	244	38	5.4	5.4	11	4	12.3	4.7
24	(P∨Q)∨R ⊢ P∨(Q∨R)	Disjunction	39	28	72	65	32	49	1.7	236	93	40	3.6	2.9	17	8	6.0	2.4
25	P∧(Q∨R) ⊢ (P∧Q)∨(P∧R)	Disjunction	44	33	75	65	39	60	1.5	322	175	54	5.0	4.5	12	7	7.3	4.0
26	(P∧Q)∨(P∧R) ⊢ P∧(Q∨R)	Disjunction	37	29	78	55	35	64	1.5	215	170	79	3.9	4.8	15	10	5.8	4.6
27	P∨(Q∧R) ⊢ (P∨Q)∧(P∨R)	Disjunction	31	26	84	44	30	68	1.4	152	124	82	3.4	4.1	17	12	4.9	4.0
28	(P∨Q)∧(P∨R) ⊢ P∨(Q∧R)	Disjunction	33	12	36	80	16	20	2.4	330	109	33	4.1	6.8	15	3	10.0	3.3
29	P→R, Q→R ⊢ (P∨Q)→R	Disjunction	27	19	70	40	24	60	1.5	98	56	57	2.4	2.3	25	15	3.6	2.1

Figure 146 Usage figures for Disjunction (conjectures 17-29)

Conjecture Number	Conjecture	Topic	Number of students attempting	Number of successful students	% of attempting students who are successful	Number of proof attempts	Number of successful proof attempts	% of proof attempts that are successful	Number of proof attempts per student	Time spent on proof attempts (mins)	Time spent on successful proofs (mins)	% of time that is spent on successful proofs	Time per proof attempt (mins)	Time per successful proof (mins)	Number of proof attempts per hour	Number of successful proofs per hour	Time per student on proof attempts (mins)	Time per student on successful proofs (mins)
30	$\neg\neg P \rightarrow P$	Negation	34	31	91	49	44	90	1.4	51	48	94	1.0	1.1	58	52	1.5	1.4
31	$P \vdash \neg\neg P$	Negation	32	24	75	50	31	62	1.6	96	60	63	1.9	1.9	31	19	3.0	1.9
32	$P \rightarrow Q \vdash \neg Q \rightarrow \neg P$	Negation	32	20	63	51	27	53	1.6	170	84	49	3.3	3.1	18	10	5.3	2.6
33	$\neg Q \rightarrow \neg P \vdash P \rightarrow Q$	Negation	21	13	62	35	20	57	1.7	103	73	71	3.0	3.6	20	12	4.9	3.5
34	$P \vee \neg P$	Negation	23	8	35	54	11	20	2.3	232	107	46	4.3	9.7	14	3	10.1	4.6
35	$P \vee Q \vdash \neg(\neg P \wedge \neg Q)$	Negation	19	8	42	38	11	29	2.0	103	36	35	2.7	3.3	22	6	5.4	1.9
36	$\neg(\neg P \wedge \neg Q) \vdash P \vee Q$	Negation	13	5	38	41	5	12	3.2	147	73	50	3.6	14.7	17	2	11.3	5.6
37	$P \wedge Q \vdash \neg(\neg P \vee \neg Q)$	Negation	11	6	55	27	8	30	2.5	110	53	49	4.1	6.7	15	4	10.0	4.9
38	$\neg(\neg P \vee \neg Q) \vdash P \wedge Q$	Negation	14	3	21	22	4	18	1.6	34	13	39	1.5	3.3	39	7	2.4	1.0
39	$\neg(P \vee Q) \vdash \neg P \wedge \neg Q$	Negation	14	8	57	18	9	50	1.3	71	34	48	3.9	3.8	15	8	5.0	2.4
40	$\neg P \wedge \neg Q \vdash \neg(P \vee Q)$	Negation	10	4	40	20	6	30	2.0	53	27	50	2.7	4.4	22	7	5.3	2.7
41	$\neg(P \wedge Q) \vdash \neg P \vee \neg Q$	Negation	8	3	38	15	3	20	1.9	43	7	16	2.9	2.4	21	4	5.4	0.9
42	$\neg P \vee \neg Q \vdash \neg(P \wedge Q)$	Negation	8	5	63	18	6	33	2.3	50	18	37	2.8	3.1	21	7	6.3	2.3
43	$\neg(P \wedge \neg P)$	Negation	14	9	64	23	13	57	1.6	18	12	66	0.8	0.9	78	44	1.3	0.8
44	$Q \rightarrow P, P \rightarrow R \vdash Q \rightarrow R$	Negation	15	13	87	17	15	88	1.1	13	10	78	0.8	0.7	79	70	0.9	0.7
45	$(P \rightarrow Q) \vee (Q \rightarrow P)$	Negation	17	2	12	30	2	7	1.8	123	16	13	4.1	8.1	15	1	7.2	0.9
46	$P \wedge \neg P \vdash Q$	Negation	13	6	46	16	8	50	1.2	17	11	67	1.1	1.4	57	28	1.3	0.9
47	$((P \rightarrow Q) \rightarrow P) \rightarrow P$	Negation	16	0	0	28	0	0	1.8	113	0	0	4.0	-	15	0	7.1	0.0

Figure 147 Usage figures for Negation (conjectures 30-47)

Conjecture Number	Conjecture	Topic	Number of students attempting	Number of successful students	% of attempting students who are successful	Number of proof attempts	Number of successful proof attempts	% of proof attempts that are successful	Number of proof attempts per student	Time spent on proof attempts (mins)	Time spent on successful proofs (mins)	% of time that is spent on successful proofs	Time per proof attempt (mins)	Time per successful proof (mins)	Number of proof attempts per hour	Number of successful proofs per hour	Time per student on proof attempts (mins)	Time per student on successful proofs (mins)
48	$\text{var } c, P(c), \forall x.(P(x) \rightarrow Q(x)) \vdash Q(c)$	Quantifiers	59	38	64	95	47	49	1.6	323	215	67	3.4	4.6	18	9	5.5	3.6
49	$\forall x.(P(x) \rightarrow Q(x)) \vdash \forall x.P(x) \rightarrow \forall x.Q(x)$	Quantifiers	78	58	74	153	67	44	2.0	1076	812	75	7.0	12.1	9	4	13.8	10.4
50	$\forall x.(P(x) \rightarrow Q(x)), \forall x.(Q(x) \rightarrow R(x)) \vdash \forall x.(P(x) \rightarrow R(x))$	Quantifiers	62	51	82	99	55	56	1.6	364	291	80	3.7	5.3	16	9	5.9	4.7
51	$\forall x.P(x) \wedge \forall x.Q(x) \vdash \forall x.(P(x) \wedge Q(x))$	Quantifiers	44	34	77	52	39	75	1.2	151	118	78	2.9	3.0	21	15	3.4	2.7
52	$\forall x.(P(x) \wedge Q(x)) \vdash \forall x.P(x) \wedge \forall x.Q(x)$	Quantifiers	39	31	79	50	35	70	1.3	332	301	91	6.6	8.6	9	6	8.5	7.7
53	$\forall x.(P(x) \rightarrow Q(x)), \exists x.P(x) \vdash \exists x.Q(x)$	Quantifiers	34	24	71	47	26	55	1.4	179	131	73	3.8	5.0	16	9	5.3	3.8
54	$\exists x.(P(x) \wedge Q(x)) \vdash \exists x.P(x) \wedge \exists x.Q(x)$	Quantifiers	30	20	67	35	22	63	1.2	125	69	56	3.6	3.2	17	11	4.2	2.3
55	$\exists x.P(x) \vee \exists x.Q(x) \vdash \exists x.(P(x) \vee Q(x))$	Quantifiers	23	15	65	24	16	67	1.0	99	89	90	4.1	5.6	15	10	4.3	3.9
56	$\exists x.(P(x) \vee Q(x)) \vdash \exists x.P(x) \vee \exists x.Q(x)$	Quantifiers	14	14	100	20	17	85	1.4	83	82	100	4.1	4.8	15	12	5.9	5.9
57	$\text{var } c, \forall x.P(x) \vdash \exists x.P(x)$	Quantifiers	16	14	88	22	16	73	1.4	70	34	48	3.2	2.1	19	14	4.4	2.1
58	$\forall x.P(x) \vdash \neg(\exists x.\neg P(x))$	Quantifiers	15	1	7	36	1	3	2.4	83	5	6	2.3	4.7	26	1	5.5	0.3
59	$\neg(\exists x.\neg P(x)) \vdash \forall x.P(x)$	Quantifiers	11	3	27	12	3	25	1.1	35	4	11	2.9	1.3	20	5	3.2	0.3
60	$\exists x.P(x) \vdash \neg(\forall x.\neg P(x))$	Quantifiers	8	2	25	9	3	33	1.1	16	3	21	1.8	1.1	34	11	2.0	0.4
61	$\neg(\forall x.\neg P(x)) \vdash \exists x.P(x)$	Quantifiers	6	2	33	6	2	33	1.0	17	6	36	2.9	3.2	21	7	2.9	1.1
62	$\neg(\forall x.P(x)) \vdash \exists x.\neg P(x)$	Quantifiers	6	1	17	6	1	17	1.0	11	2	22	1.8	2.3	34	6	1.8	0.4
63	$\exists x.\neg P(x) \vdash \neg(\forall x.P(x))$	Quantifiers	6	0	0	7	0	0	1.2	8	0	0	1.2	-	51	0	1.4	0.0
64	$\neg(\exists x.P(x)) \vdash \forall x.\neg P(x)$	Quantifiers	3	2	67	3	2	67	1.0	3	3	95	0.9	1.3	67	45	0.9	0.9
65	$\forall x.\neg P(x) \vdash \neg(\exists x.P(x))$	Quantifiers	4	0	0	4	0	0	1.0	12	0	0	3.0	-	20	0	3.0	0.0

Figure 148 Usage figures for Quantifiers (conjectures 48-65)

Conjecture Number	Conjecture	Topic	Number of students attempting	Number of proof attempts	Number of successful proof attempts
66	$\forall x.P(x) \vdash \exists x.P(x)$ NOT	False	2	2	0
67	$P(c), \forall x.(P(x) \rightarrow Q(x)) \vdash Q(c)$ NOT	False	1	1	0
68	$\text{var } c, Q(c) \vdash \forall x.(P(x) \rightarrow Q(x))$ NOT	False	4	4	0
69	$(\forall x.P(x)) \rightarrow (\forall x.Q(x)) \vdash \forall x.(P(x) \rightarrow Q(x))$ NOT	False	3	3	0
70	$(\exists x.P(x)) \wedge (\exists x.Q(x)) \vdash \exists x.(P(x) \wedge Q(x))$ NOT	False	4	8	0
101	$\neg(P \wedge \neg Q) \vdash P \rightarrow Q$	User-entered	1	1	1
102	$\forall x.(P(x) \rightarrow Q(x)), \forall x.(Q(x) \rightarrow \neg R(x)) \vdash \forall x.(P(x) \rightarrow \neg R(x))$	User-entered	2	2	1
103	$(P \vee Q) \wedge (P \vee R) \vdash P \wedge (Q \wedge R)$	User-entered	2	2	0
104	$\forall x.(P(x) \rightarrow Q(x)), \forall x.(P(x) \rightarrow R(x)) \vdash \forall x.(P(x) \rightarrow (Q(x) \wedge R(x)))$	User-entered	1	1	1
105	$\exists x.(P(x) \wedge Q(x)), \neg(\exists x.(Q(x) \wedge R(x))) \vdash \exists x.(P(x) \wedge \neg R(x))$	User-entered	1	1	0
106	$\forall x.(P(x) \rightarrow \neg Q(x)), \exists x.(P(x) \wedge R(x)) \vdash \exists x.(R(x) \wedge \neg Q(x))$	User-entered	1	1	1
107	$\forall x.(P(x) \rightarrow Q(x)), \exists x.P(x) \vdash \exists x.(P(x) \wedge Q(x))$	User-entered	1	2	1
108	$\forall x.(P(x) \rightarrow \neg Q(x)), \forall x.(R(x) \rightarrow P(x)) \vdash \forall x.(R(x) \rightarrow \neg Q(x))$	User-entered	1	1	1
109	$\forall x.P(x) \vee \forall x.Q(x) \vdash \forall x.(P(x) \vee Q(x))$	User-entered	1	1	1
110	$\neg P \rightarrow P \vdash P$	User-entered	1	1	0
111	$\exists x.(P(x) \wedge Q(x)), \forall x.(Q(x) \rightarrow \neg R(x)) \vdash \forall x.(R(x) \rightarrow \neg Q(x))$	User-entered	1	1	0
112	$\exists x.(P(x) \wedge Q(x)), \forall x.(Q(x) \rightarrow \neg R(x)) \vdash \forall x.(P(x) \rightarrow \neg R(x))$	User-entered	1	1	0
113	$P \rightarrow Q, R \rightarrow S \vdash (P \wedge R) \rightarrow (Q \wedge S)$	User-entered	1	1	1
114	$\exists x.(P(x) \rightarrow \forall x.P(x))$	User-entered	1	1	0
115	$(P \vee Q) \wedge (P \vee R) \vdash P \wedge (Q \vee R)$	User-entered	1	1	0
116	$\exists x.(P(x) \wedge Q(x)), \exists x.(Q(x) \wedge R(x)) \vdash \exists x.(P(x) \wedge \neg R(x))$	User-entered	1	1	0
117	$R \vee S \vdash R$	User-entered	1	1	0

Figure 149: Usage figures for the False and User-Entered conjectures

8.5 Survey2

Tools for Learning Logic: Second Survey

This survey follows up the one you completed at the beginning of term. It is completely confidential. We ask for your name only so that we can connect the two surveys.

Thank you once again for your help.

James Aczel & Pat Fung, Open University

Section 1: Experience of the Course

Family Name: _____ *First Name:* _____

*In your opinion, how **worthwhile** do you think the "Introduction to Logic" course has been?*

very worthwhile	☐	**Please tick one box only**
fairly worthwhile	☐	
slightly worthwhile	☐	
not worthwhile at all	☐	

*How **difficult** have you found the logic course?*

difficult	☐	**Please tick one box only**
fairly difficult	☐	
fairly easy	☐	
easy	☐	

*How **interesting** was the logic course for you?*

interesting	☐	**Please tick one box only**
fairly interesting	☐	
fairly uninteresting	☐	
uninteresting	☐	

Survey2 (continued)

Section 2: Experience of Jape

For about how many hours would you estimate you used Jape? ☐

*What did you most **like** about using Jape?*

1.

2.

3.

*What did you most **dislike** about using Jape?*

1.

2.

3.

Did you make use of the printing facility?

often ☐
sometimes ☐
never ☐

Please tick one box only

*Overall, how **helpful** has Jape been for you in learning logic?*

very helpful ☐
fairly helpful ☐
slightly helpful ☐
not helpful at all ☐
harmful ☐

Please tick one box only

How would you improve Jape?

8.6 Prior Proofs

These are proofs that students may have met when using ItL Jape, when studying the lecture notes, when reading the Jape manual, and when taking part in the Reflection Study. 'E' refers to a conjecture that has been set as an exercise, rather than proved as an example. '(a)' and '(b)' refer to a proof that comes in two parts.

Jape	Lecture	Manual	Study	Implication
1	1	y		P, P → Q ⊢ Q
2	2	y	4	P → Q, Q → R , P ⊢ R
3	3		1	P → (Q → R), P → Q, P ⊢ R
4	4		2	P → Q, Q → R ⊢ P → R
5	5E	y	3	P → (Q → R) ⊢ Q → (P → R)
6	6E			P → (Q → R) ⊢ (P → Q) → (P → R)
7	7			P ⊢ Q → P
8	8			⊢ P → (Q → P)
	9			⊢ P → P
	10			P ⊢ P
9	11E			P → Q ⊢ (Q → R) → (P → R)
10				P → (Q → (R → S)) ⊢ R → (Q → (P → S))
11				⊢ (P → (Q → R)) → ((P → Q) → (P → R))
			3b	P → Q, P ⊢ R → Q
			5	P → Q, R → (S → R), R, S → P, R → P ⊢ Q
Jape	**Lecture**	**Manual**	**Study**	**Conjunction**
12				P, Q ⊢ P ∧ Q
13			6	P ∧ Q ⊢ P
14				P ∧ Q ⊢ Q
	12			P ⊢ (P ∧ P) ∧ P
	13			(P ∧ Q) ∧ R ⊢ Q
	14			(P ∧ Q) ∧ (R ∧ S) ⊢ Q ∧ R
15	15			(P ∧ Q) → R ⊢ P → (Q → R)
16	16		10	P → (Q → R) ⊢ (P ∧ Q) → R
	17			[15 and 16]
			7	(P → Q) ∧ (Q → R), Q ⊢ R
			8	P, R ⊢ P ∧ (Q → R)
			9	P, Q, (P ∧ Q) → R ⊢ R
		y		P → (Q ∧ R), P ⊢ Q
		y		P ∧ (Q ∧ R) ⊢ (P ∧ Q) ∧R
Jape	**Lecture**	**Manual**	**Study**	**Disjunction**
17			12	P ⊢ P ∨ Q
18				Q ⊢ P ∨ Q
	18			P ⊢ (Q ∨ P) ∨ R
	19			P ⊢ (P ∨ Q) ∧ (Q ∨ P)
19	20		13	P ∨ Q ⊢ Q ∨ P
20	21		14	Q → R ⊢ (P ∨ Q) → (P ∨ R)
21	22(a)			P ∨ P ⊢ P
22	22(b)			P ⊢ P ∨ P
23	25E(a)			P ∨ (Q ∨ R) ⊢ (P ∨ Q) ∨ R
24	25E(b)			(P ∨ Q) ∨ R ⊢ P ∨ (Q ∨ R)
25	23E(a)			P ∧ (Q ∨ R) ⊢ (P ∧ Q) ∨ (P ∧ R)
26	23E(b)			(P ∧ Q) ∨ (P ∧ R) ⊢ P ∧ (Q ∨ R)
27	24E(a)		11	P ∨ (Q ∧ R) ⊢ (P ∨ Q) ∧ (P ∨ R)
28	24E(b)	y		(P ∨ Q) ∧ (P ∨ R) ⊢ P ∨ (Q ∧ R)
29	26E			P → R, Q → R ⊢ (P ∨ Q) → R
			15	Q, (Q → R) ∨ P ⊢ P ∨ (Q → R)
			16	((P → S) ∧ Q) ∨ ((P → S) ∧ R) ⊢ P → S
			17	Q → R ⊢ (R ∨ Q) → (P ∨ R)

Prior proofs (continued)

Jape	Lecture	Manual	Study	Negation
30	27			⊢ ¬¬P → P
31	28			P ⊢ ¬ ¬P
32	29		20	P → Q ⊢ ¬Q → ¬P
33	30		21	¬Q → ¬P ⊢ P → Q
34	31	y	23	⊢ P ∨ ¬P
35	32(a)		26	P ∨ Q ⊢ ¬(¬P ∧ ¬Q)
36	32(b)		22	¬(¬P ∧ ¬Q) ⊢ P ∨ Q
37				P ∧ Q ⊢ ¬(¬P ∨ ¬Q)
38				¬(¬P ∨ ¬Q) ⊢ P ∧ Q
39		y		¬(P ∨ Q) ⊢ ¬P ∧ ¬Q
40				¬P ∧ ¬Q ⊢ ¬(P ∨ Q)
41				¬(P ∧ Q) ⊢ ¬P ∨ ¬Q
42				¬P ∨ ¬Q ⊢ ¬(P ∧ Q)
43	33			⊢ ¬(P ∧ ¬P)
	34			Q → R ⊢ ¬¬(Q → R)
44	35			Q → P, P → R ⊢ Q → R
45	36	y	25	⊢ (P → Q) ∨ (Q → P)
46	37			P ∧ ¬P ⊢ Q
47	38		27	⊢ ((P → Q) → P) → P
		y		¬¬P ⊢ Q → P
		y		P → Q ⊢ ¬(P ∧ ¬Q)
				(Q ∧ P) ∨ (Q → R) ⊢ ¬¬((Q ∧ P) ∨ (Q → R))
			18	P → Q, ¬¬P ⊢ Q
			19	P → Q, ¬Q ⊢ ¬P
			24	⊢ ¬(¬P ∧ ¬Q) → P ∨ Q
Jape	**Lecture**	**Manual**	**Study**	**Quantifiers**
48	39	y	28	var c, P(c), ∀*x*.(P(*x*) → Q(*x*)) ⊢ Q(c)
49	40	y	30	∀*x*.(P(*x*) → Q(*x*)) ⊢ ∀*x*.P(*x*) → ∀*x*.Q(*x*)
50	41		29	∀*x*.(P(*x*) → Q(*x*)), ∀*x*.(Q(*x*) → R(*x*)) ⊢ ∀*x*.(P(*x*) → R(*x*))
51	43			∀*x*.P(*x*) ∧ ∀*x*.Q(*x*) ⊢ ∀*x*.(P(*x*) ∧ Q(*x*))
52	42			∀*x*.(P(*x*) ∧ Q(*x*)) ⊢ ∀*x*.P(*x*) ∧ ∀*x*.Q(*x*)
	44		31	∀*x*.P(*x*) ∨ ∀*x*.Q(*x*) ⊢ ∀*x*.(P(*x*) ∨ Q(*x*))
53	46		34	∀*x*.(P(*x*) → Q(*x*)), ∃*x*.P(*x*) ⊢ ∃*x*.Q(*x*)
54	47	y		∃*x*.(P(*x*) ∧ Q(*x*)) ⊢ ∃*x*.P(*x*) ∧ ∃*x*.Q(*x*)
55	48			∃*x*.P(*x*) ∨ ∃*x*.Q(*x*) ⊢ ∃*x*.(P(*x*) ∨ Q(*x*))
56	49		37	∃*x*.(P(*x*) ∨ Q(*x*)) ⊢ ∃*x*.P(*x*) ∨ ∃*x*.Q(*x*)
57	45			var c, ∀*x*.P(*x*) ⊢ ∃*x*.P(*x*)
58	50			∀*x*.P(*x*) ⊢ ¬(∃*x*. ¬P(*x*))
59	51			¬(∃*x*. ¬P(*x*)) ⊢ ∀*x*.P(*x*)
60	52			∃*x*.P(*x*) ⊢ ¬(∀*x*. ¬P(*x*))
61	53			¬(∀*x*. ¬P(*x*)) ⊢ ∃*x*.P(*x*)
62				¬(∀*x*.P(*x*)) ⊢ ∃*x*. ¬P(*x*)
63				∃*x*. ¬P(*x*) ⊢ ¬(∀*x*.P(*x*))
64				¬(∃*x*.P(*x*)) ⊢ ∀*x*. ¬P(*x*)
65				∀*x*. ¬P(*x*) ⊢ ¬(∃*x*.P(*x*))
			32	*Misproof* ∀*x*.(P(*x*) ∨ Q(*x*)) ⊢ ∀*x*.P(*x*) ∨ ∀*x*.Q(*x*)
			33	*Misproof* P(m), ∀*x*.(P(*x*) → Q(*x*)) ⊢ ∀*x*.Q(*x*)
			35	var c, P(c) ⊢ ∀*x* P(*x*) ∨ ∃*x* P(*x*)
		y		∃*x*. ∀*y*. P(*x*, *y*) ⊢ ∀*y*. ∃*x*. P(*x*, *y*)
Jape	**Lecture**	**Manual**	**Study**	**Misproofs**
66		y		∀*x*.P(*x*) ⊢ ∃*x*.P(*x*)
67		y		P(c), ∀*x*.(P(*x*) → Q(*x*)) ⊢ Q(c)
68				var c, Q(c) ⊢ ∀*x*.(P(*x*) → Q(*x*))
69				∀*x*.P(*x*) → ∀*x*.Q(*x*) ⊢ ∀*x*.(P(*x*) → Q(*x*))
70			36	∃*x*.P(*x*) ∧ ∃*x*.Q(*x*) ⊢ ∃*x*.(P(*x*) ∧ Q(*x*))

8.7 Conjectures used in the Reflection Study

Implication Conjectures used in the Reflection Study

Proof 1 Strategies for →E forwards

1: P→(Q→R), P→Q, P premises
…
2: R

Proof 2 Strategies for →I backwards

1: P→Q, Q→R premises
…
2: P→R

Proof 3 Forward-fixated reasoning - →E forwards versus I backwards

1: P→(Q→R) premise
…
2: Q→(P→R)

Proof 3b Strategies for →E forwards & →I backwards

1: P→Q, P premises
…
2: R→Q

Proof 4 (Jape) Understanding of unify / hyp after →E backwards

1: P→Q, Q→R, P premises
…
2: _A
…
3: _A→R
4: R →E 2, 3

Proof 5 Incomplete forward reasoning for →E forwards

1: P→Q, R→(S→R), R, S→P, R→P premises
…
2: Q

Conjunction Conjectures used in the Reflection Study

Proof 6 Strategies for ∧E forwards

1: P∧Q premise
…
2: P

Proof 7 Strategies for ∧E forwards (2)

1: (P→Q)∧(Q→R), Q premises
…
2: R

Proof 8 Strategies for ∧I backwards

1: P, R premises
…
2: P∧(Q→R)

Proof 9 The direction of ∧I

1: P, Q, (P∧Q)→R Premises
…
2: R

Proof 10 Forward-fixated reasoning - →E forwards versus →I backwards

1: P→(Q→R) premise
…
2: (P∧Q)→R

Disjunction Conjectures used in the Reflection Study

Proof 11 Strategies for ∨E forwards

1: P ∨ (Q∧R) premise
…
2: (P∨Q) ∧ (P∨R)

Proof 12 Strategies for ∨I backwards

1: P premise
…
2: P∨Q

Proof 13 Checking for provability - ∨E forwards versus ∨I backwards

1: P∨Q premise
…
2: Q∨P

Proof 14 Forward-fixated reasoning - →E forwards versus →I backwards

1: Q→R premise
…
2: (P∨Q)→(P∨R)

Proof 15 Sensible use of →E

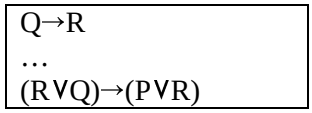
1: Q, (Q→R)∨P premises
…
2: P∨(Q→R)

Proof 16 ∨E forwards versus →I backwards

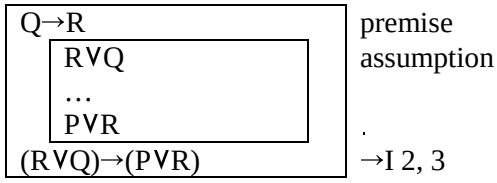
1: ((P→S)∧Q) ∨ ((P→S)∧R) premise
…
2: P→S

Proof 17 “Looking for blank justifications” versus “breaking down”

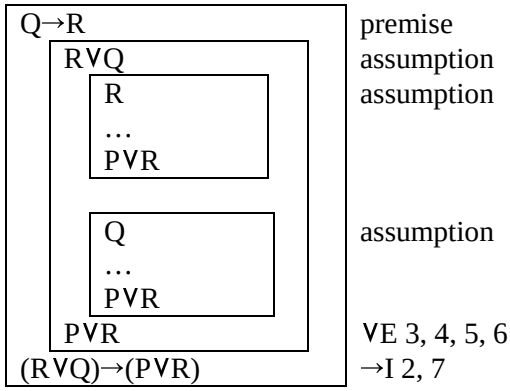
8.1.1 Strategies for →I



8.1.2 Strategies for ∨E



8.1.3 ∨I versus →E



Response	*Suggests*
∨I on line 4	A strategy of focusing on the first line without a justification.
→E on line 5	A strategy of focusing on the first line that hasn’t been broken down.

Negation Conjectures used in the Reflection Study

Proof 18 Strategies for ¬E forwards

1: P→Q, ¬¬P premises
…
2: Q

Proof 19 Strategies for ¬I backwards

1: P→Q, ¬Q premises
…
2: ¬P

Proof 20 Strategies for ¬I backwards (after →I backwards)

1: P→Q premise
…
2: ¬Q→¬P

Proof 21 Strategies for ¬E¬I backwards (A⊢B case)

1: ¬Q→¬P premise
…
2: P→Q

Proof 22 Strategies for ¬E¬I backwards (¬A⊢B and ¬A⊢¬B cases)

1: ¬(¬P∧¬Q) premise
…
2: P∨Q

Proof 23 Strategies for ¬E¬I backwards (⊢B case)

…
1: P∨¬P

Proof 24 Sequent Introduction using ¬(¬P∧¬Q) ⊢ P∨Q

…
1: ¬(¬P∧¬Q) → P∨Q

Proof 25 Sequent Introduction using ⊢P∨¬P

1: …
(P→Q) ∨ (Q→P)

Proof 26 ∨E forwards versus ¬I backwards

1: P∨Q
…
2: ¬(¬P∧¬Q)

Proof 27 Pierce's Law

1: …
((P→Q)→P)→P

Quantifiers Conjectures used in the Reflection Study

Proof 28 Strategies for ∀E forwards (in the Socrates syllogism)

1:	var c, P(c), ∀*x* (P(*x*)→Q(*x*))	premises
	…	
2:	Q(c)	

Proof 29 Strategies for ∀I backwards

1:	∀*x* (P(*x*)→Q(*x*)), ∀*x* (Q(*x*)→R(*x*))	premises
	…	
2:	∀*x* (P(*x*)→R(*x*))	

Proof 30 Strategies for →I backwards, ∀I backwards and ∀E forwards

1:	∀*x* (P(*x*)→Q(*x*))	premise
	…	
2:	∀*x* P(*x*) → ∀*x* Q(*x*)	

Proof 31 Consideration of efficiency in a ∀ proof

1:	∀*x* P(*x*)∨ ∀*x* Q(*x*)	premise
	…	
2:	∀*x* (P(*x*)∨ Q(*x*))	

Proof 32 Checking for provability in a misproof involving ∀-formulae

1:	∀*x* (P(*x*)∨Q(*x*))	premise
	…	
2:	∀*x* P(*x*) ∨ ∀*x* Q(*x*)	

Proof 33 Misproof because of variable scope

1:	P(m), ∀*x* (P(*x*)→Q(*x*))	premises
2:	P(m)→Q(m)	∀E 1.2
	…	
3:	Q(m)	
4:	∀*x* Q(*x*)	∀I 2, 3

Proof 34 ∃E forwards versus ∀E forwards and ∃I backwards

1:	∀*x* (P(*x*)→Q(*x*)), ∃*x* P(*x*)	premises
	…	
2:	∃*x* Q(*x*)	

Proof 35 Use of ∨I then ∃I backwards

1:	var c, P(c)	premises
	…	
2:	∀*x* P(*x*) ∨ ∃*x* P(*x*)	

Proof 36 ∃E Misproof

1:	∃*x* P(*x*) ∧ ∃*x* Q(*x*)	premise
2:	∃*x* P(*x*)	∧E 1
3:	∃*x* Q(*x*)	∧E 1
4:	P(m)	
	…	
5:	∃*x* (P(*x*)∧Q(*x*))	
6:	∃*x* (P(*x*)∧Q(*x*))	∃E 2, 4, 5

Proof 37 Choice between ∃E forwards and ∃I backwards

1:	∃*x* (P(*x*) ∨ Q(*x*))	premise
	…	
2:	∃*x* P(*x*) ∨ ∃*x* Q(*x*)	